Unit 2 Review

Statics

Statics Principles

The laws of motion describe the interaction of forces acting on a body

– Newton's First Law of Motion *(law of inertia):*

An object in a state of rest or uniform motion will continue to be so unless acted upon by another force.

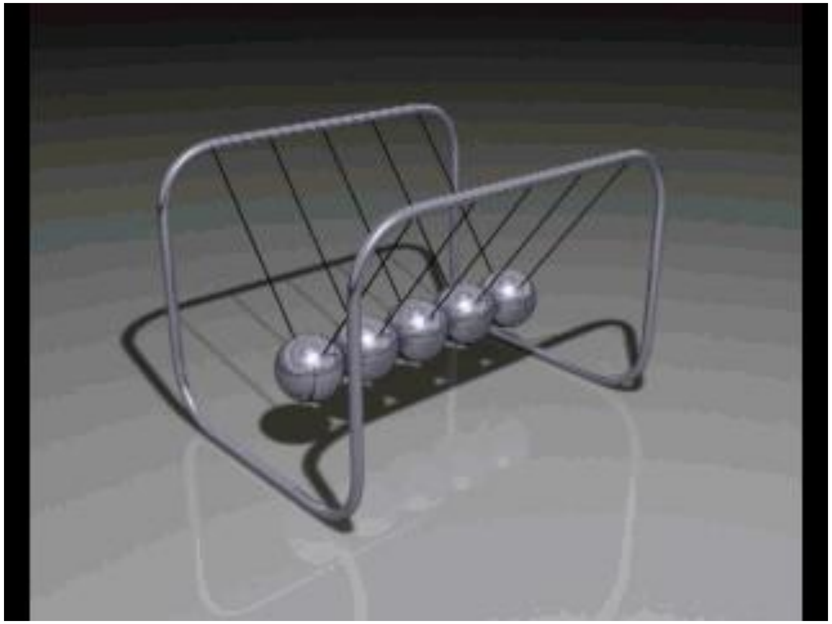
– Newton's Second Law of Motion:

Force = Mass x Acceleration

Statics Principles

Newton's Third Law of Motion:

For every action force, there is an equal and opposite reaction force.



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Equilibrium

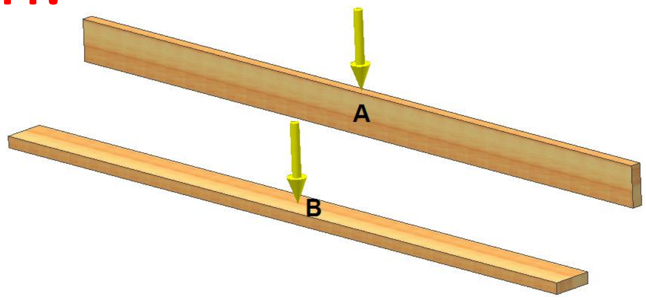
Static Equilibrium:

A condition where there are no net external forces acting upon a particle or rigid body and the body remains at rest or continues at a constant velocity

SUM OF ALL FORCES EQUALS ZERO

Structural Member Properties

- Centroid: center of gravity or center of mass. Object is in state of equilibrium if balanced along its centroid
- Moment of Inertia: Stiffness of an object related to its shape. a higher Moment of Inertia produces a greater resistance to deformation.

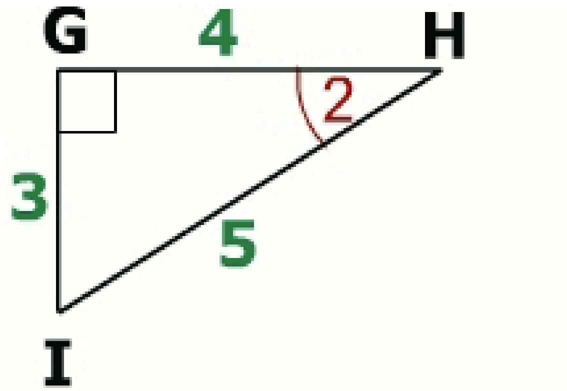


- Modulus of Elasticity

Ratio of stress to strain. Inherent to the material.

Right Triangle Review

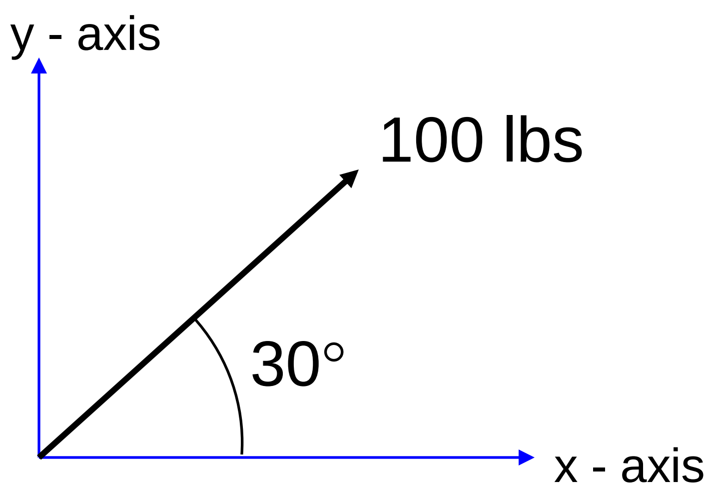
SOHCAHTOA



Sin θ = O/H
Cos θ = A/H
Tan θ = O/A

Be able to use Right triangle properties or Pythagorean's Theorem to solve for a hypotenuse

Vectors: have magnitude, direction and sense



The vector has a magnitude of 100 lbs, a direction of 30 degrees CCW from the positive x axis. Its sense is up and to the right.

$F_x = F * \cos\theta$

$F_x = 100\text{lbs} * \cos 30$

$F_x = 87$ lbs

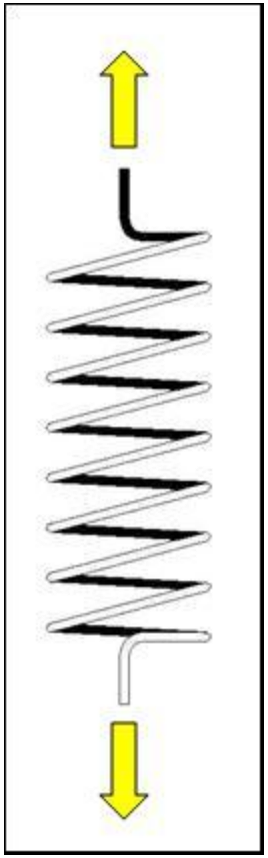
$F_y = F * \sin\theta$

$F_y = 100\text{lbs} * \sin 30$

$F_y = 50$ lbs

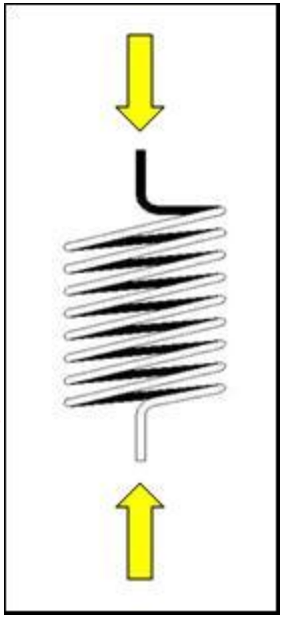
Forces in Tension and Compression

A force is a push or pull exerted by one object on another.



A tensile force expands or lengthens the object it is acting on.

A compressive force compresses or shortens the object it is acting on.



Moments

A *moment* of a force is a measure of its tendency to cause a body to rotate about a point or axis.

It is the same as torque.

A moment (M) is calculated using the formula:

Moment = Force * Distance

M = F * D

Always use the perpendicular distance between the force and the point!

Moments

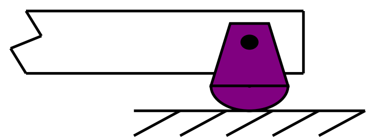
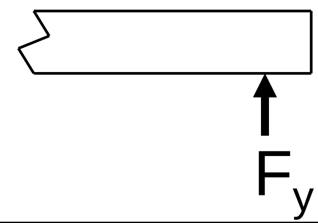
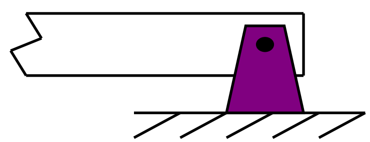
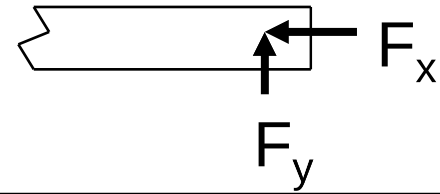
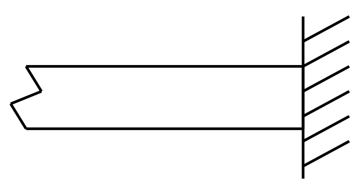
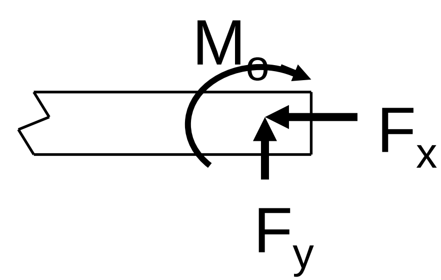
Typically it is assumed:

- A moment with a tendency to rotate counter clockwise (CCW) is considered to be a positive moment.

- A moment with a tendency to rotate clockwise (CW) is considered to be a negative moment.

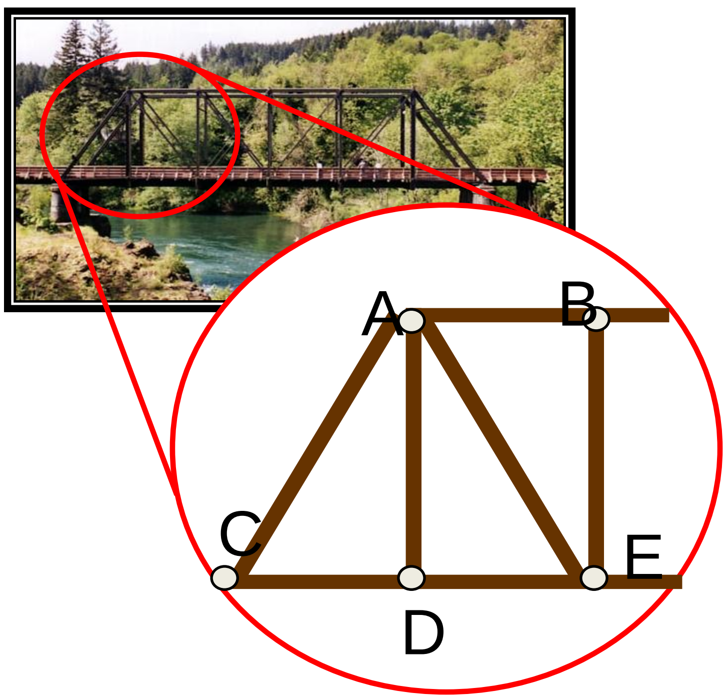
Force/Free Body Diagrams

FBDs are used to illustrate and calculate forces acting upon a given body.

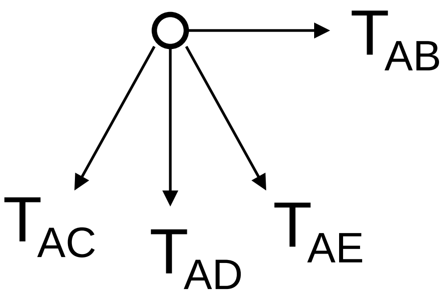
Roller:		
Pin Connection:		
Fixed Support:		

Force/Free Body Diagrams

Draw a FBD of the pin at point A:



Pin-Connected Pratt Through Truss Bridge



Free Body Diagram of pin A

(If you consider the third dimension, then there is an additional force acting on point A into the paper: The force of the beam that connects the front of the bridge to the back of the bridge.)

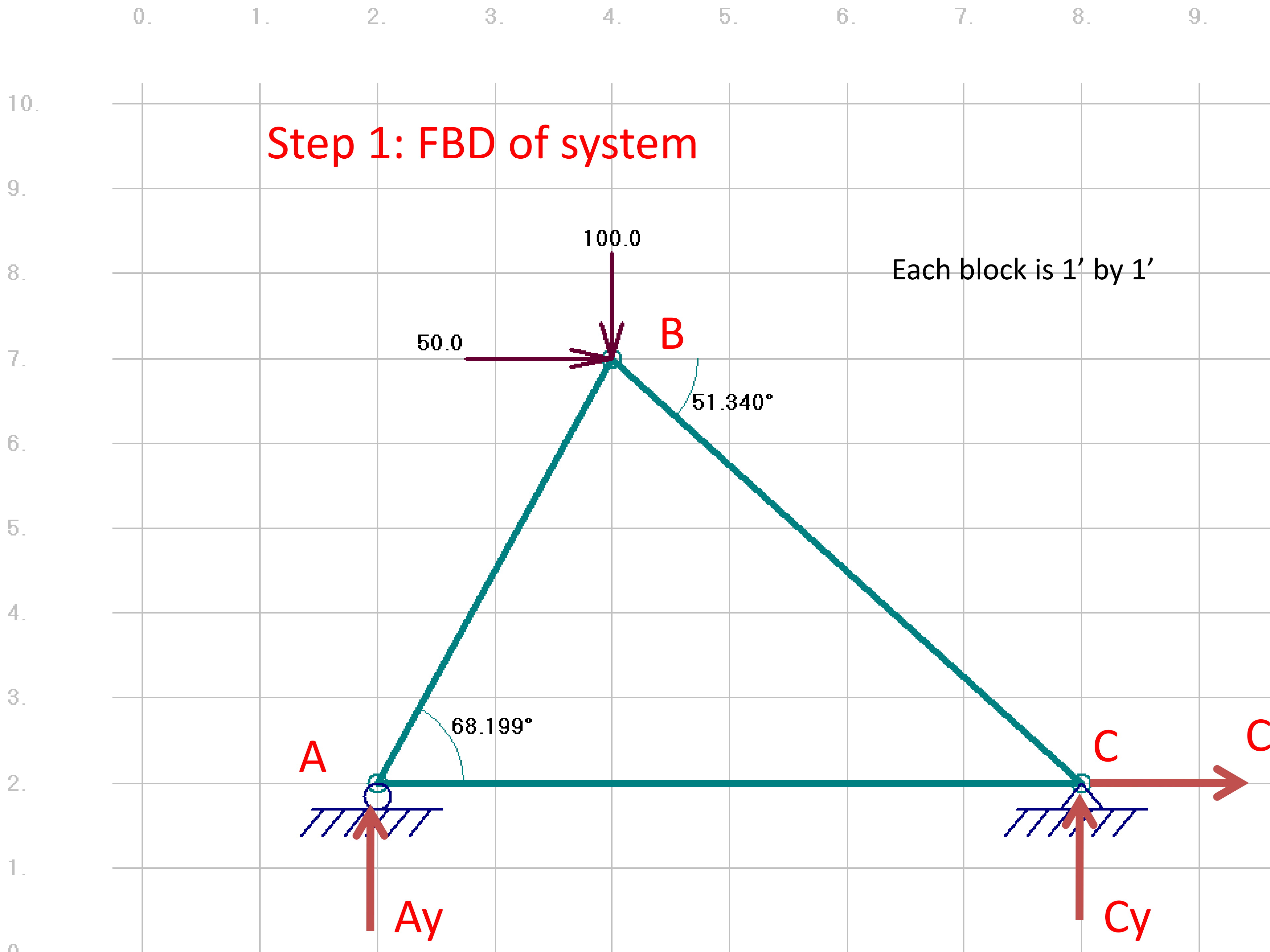
Steps for finding Reaction Forces

1. Draw a FBD of the entire system
2. $\Sigma F_X = 0$
3. $\Sigma F_Y = 0$
4. $\Sigma M = 0$
5. Use the above equations to solve for reaction forces (substitute back into 2 or 3)
6. Redraw the FBD with reaction forces

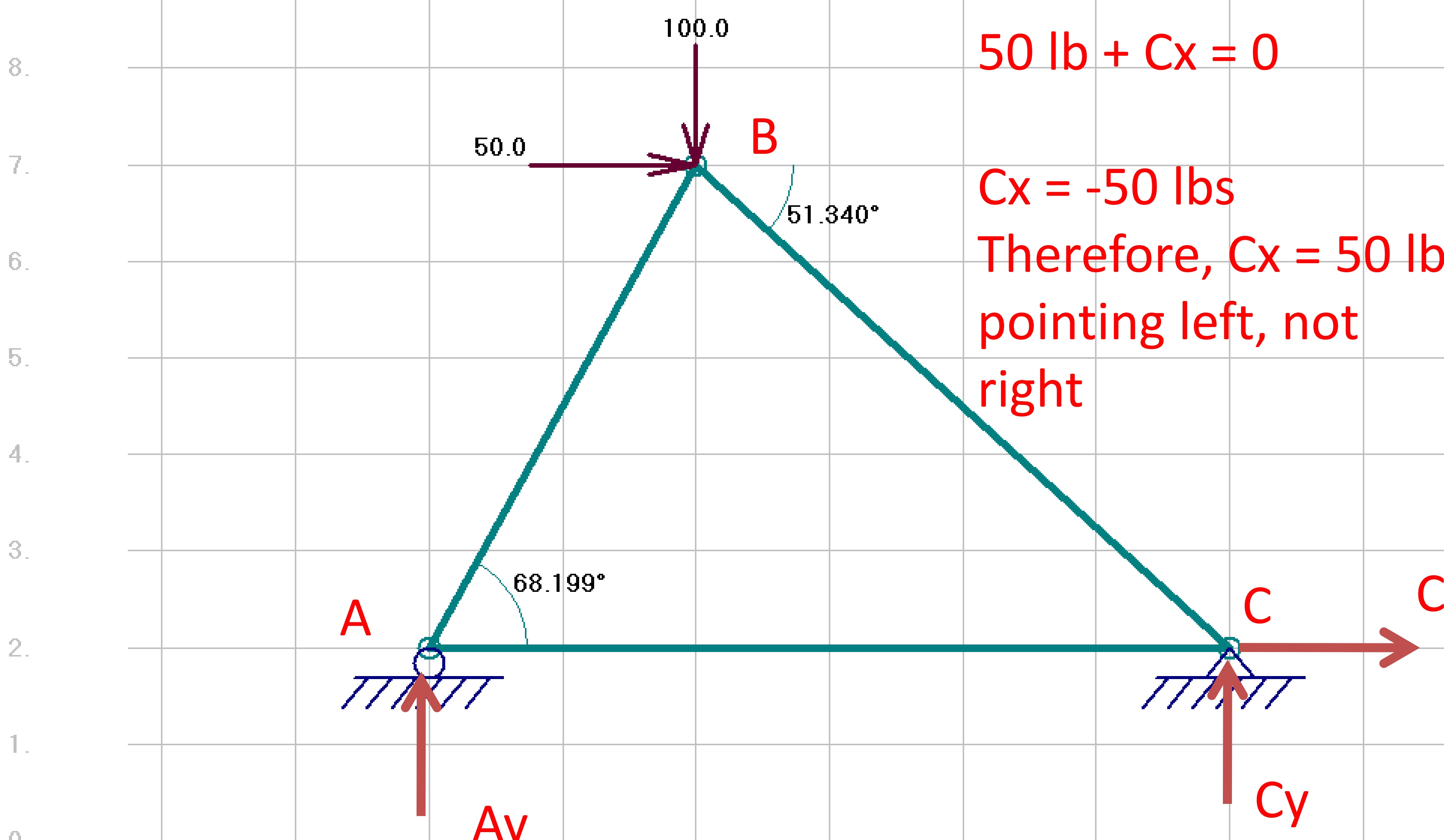
You may need to sum moments about more than 1 point

Step 1: FBD of system

Each block is 1' by 1'



Step 2: Sum Forces in X direction = to zero



50 lb + Cx = 0

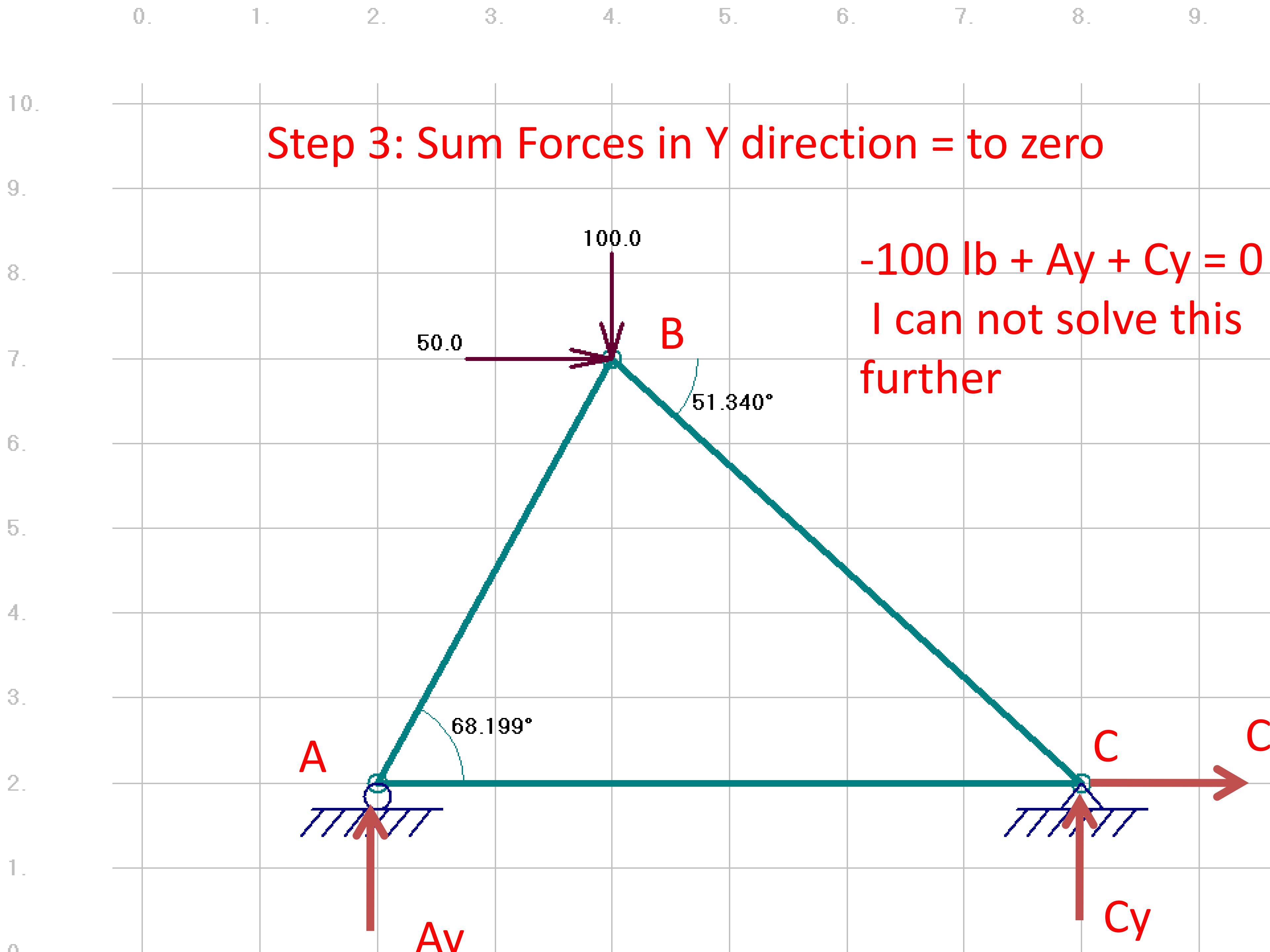
Cx = -50 lbs

Therefore, Cx = 50 lb pointing left, not right

Step 3: Sum Forces in Y direction = to zero

$-100 \text{ lb} + Ay + Cy = 0$

I can not solve this further



Step 4: Sum Moments = to zero

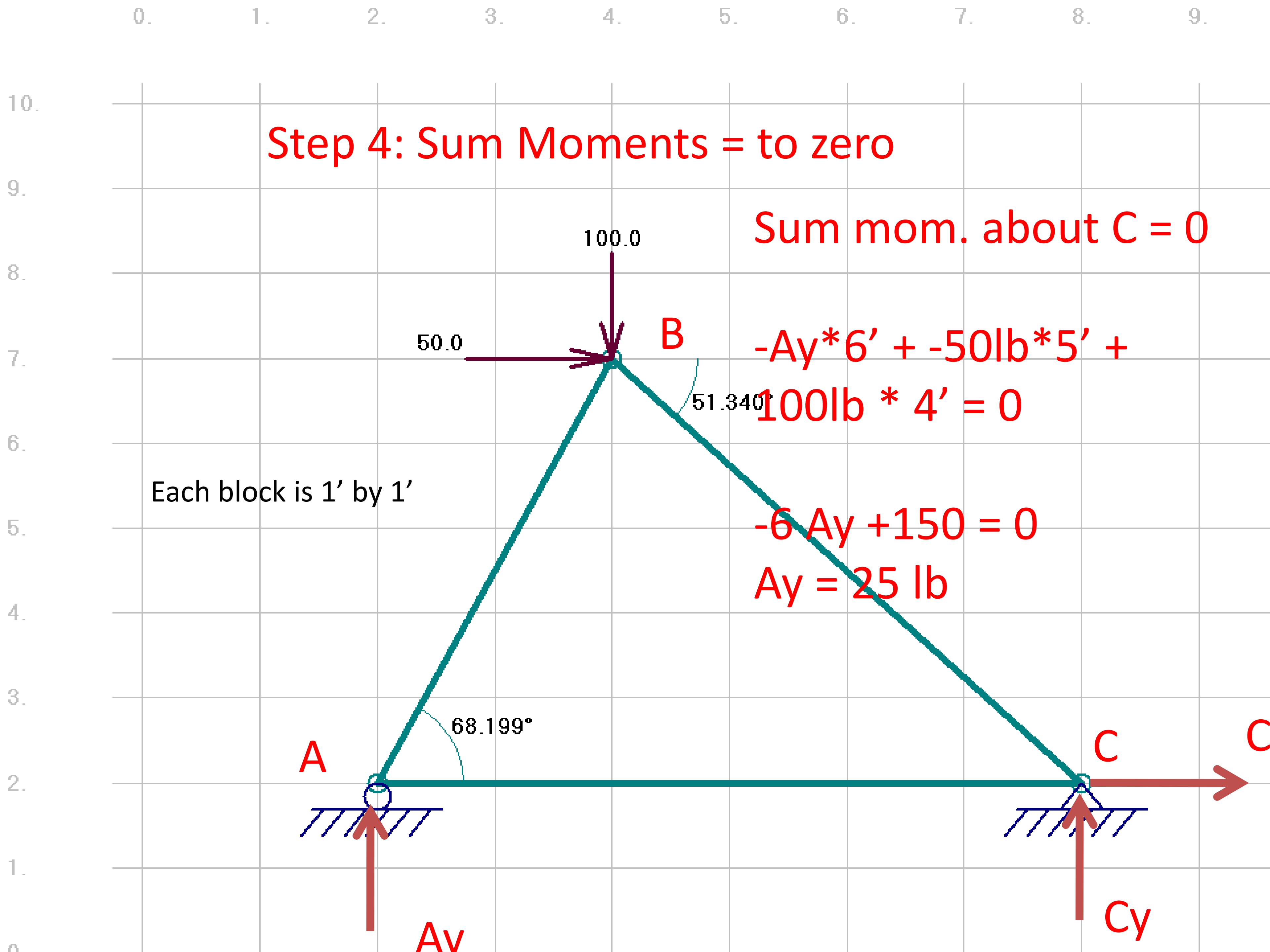
Sum mom. about C = 0

-Ay*6’ + -50lb*5’ + 100lb * 4’ = 0

-6 Ay +150 = 0

Ay = 25 lb

Each block is 1’ by 1’

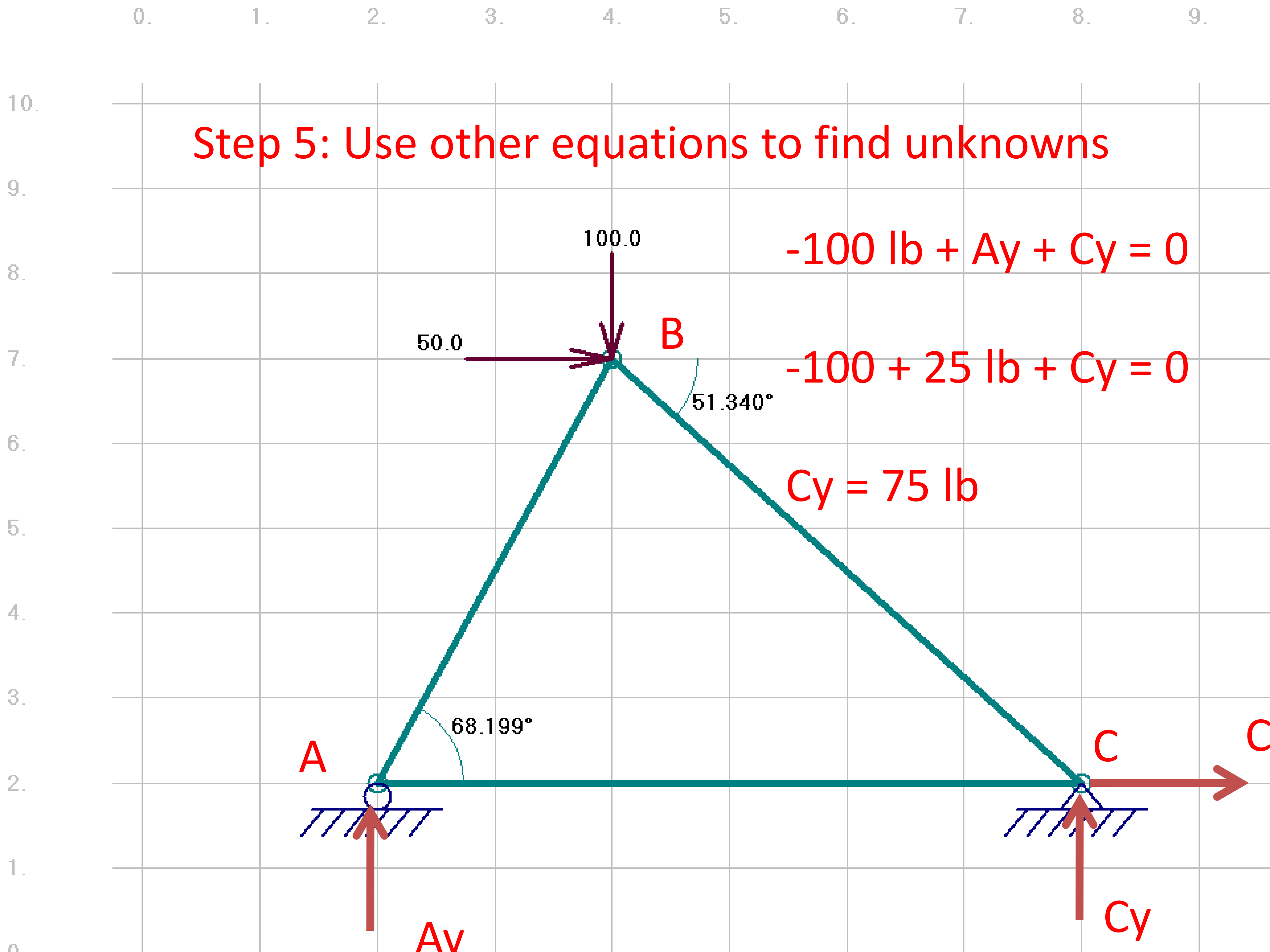


Step 5: Use other equations to find unknowns

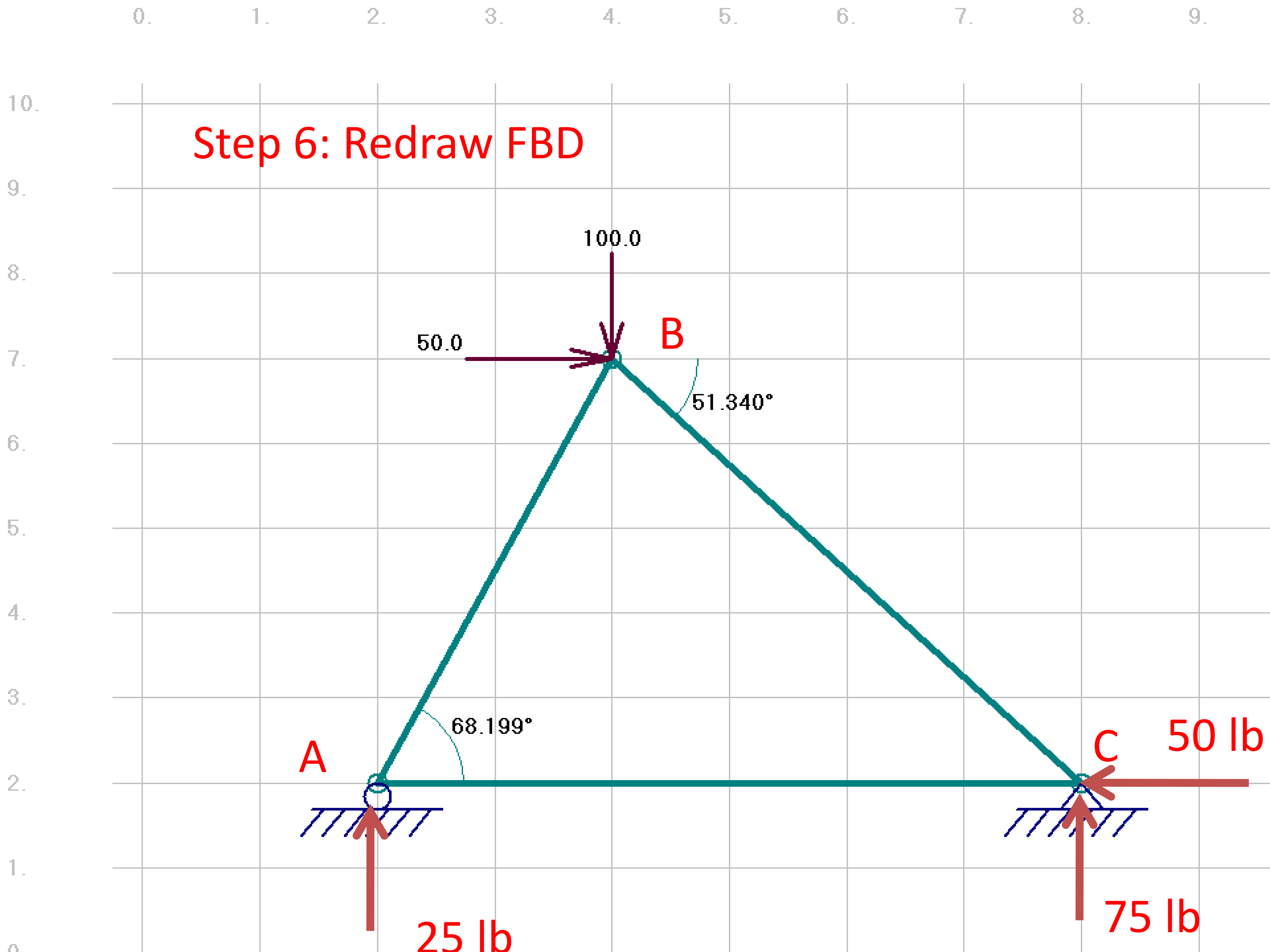
$-100\text{ lb} + Ay + Cy = 0$

$-100 + 25\text{ lb} + Cy = 0$

$Cy = 75\text{ lb}$



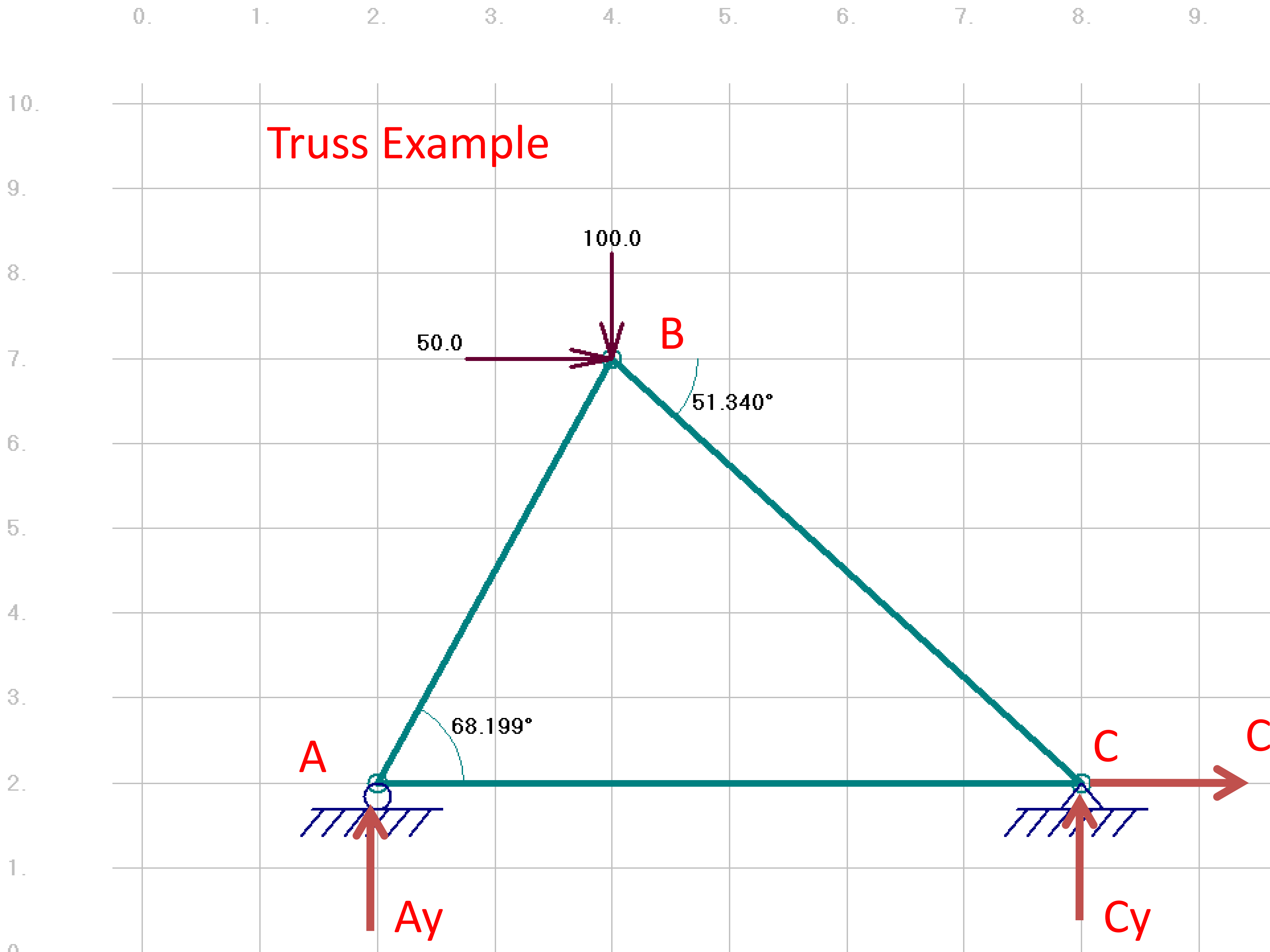
Step 6: Redraw FBD



Truss Review

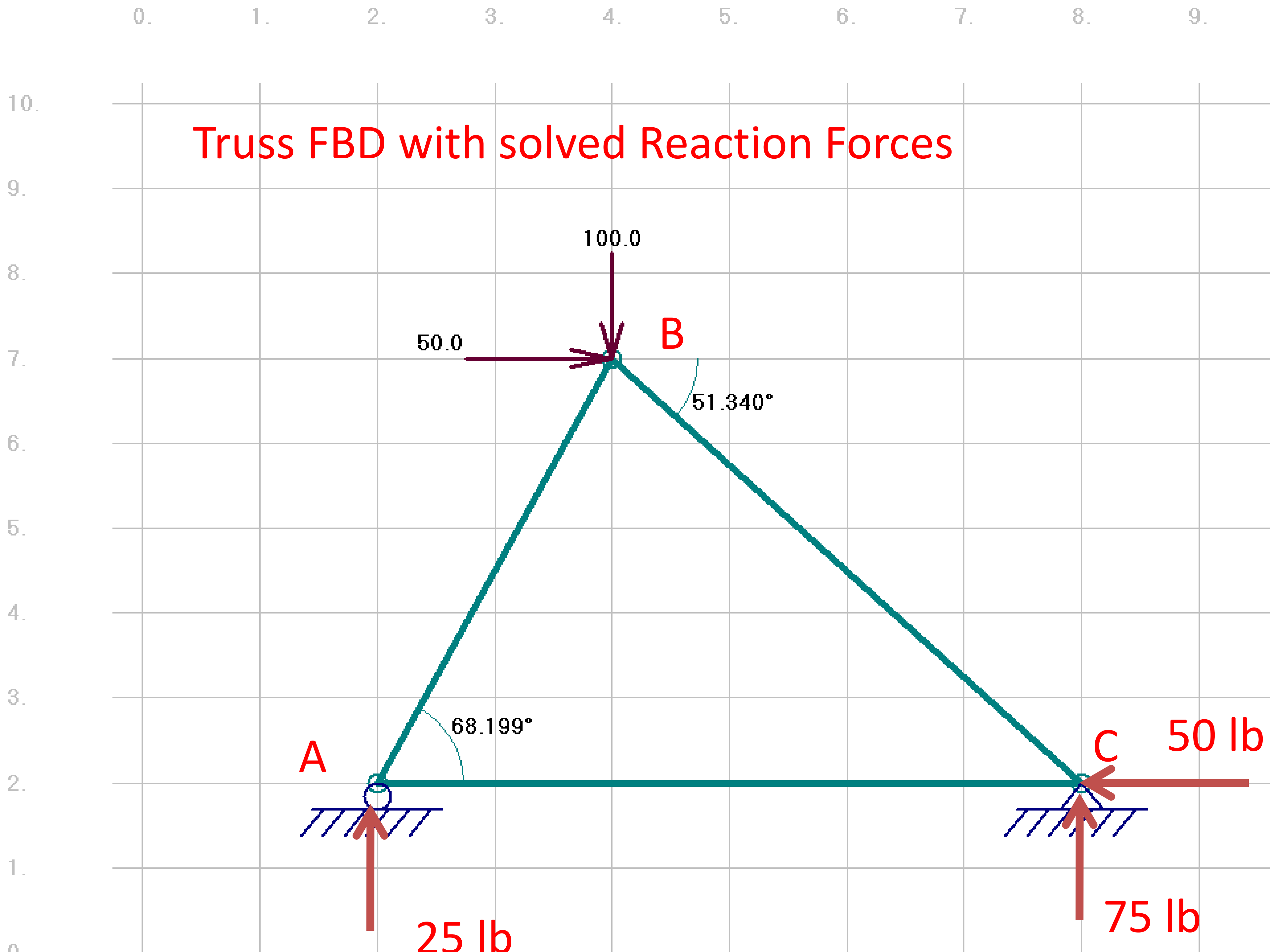
Steps for finding Truss Forces

1. Solve for Reaction forces
   a. Draw a FBD of the entire system
   b. $\Sigma F_X = 0; \Sigma F_Y = 0; \Sigma M = 0$
   c. Use the above equations to solve for reaction forces
2. FBD of each joint (use vector properties)
3. $\Sigma F_X = 0; \Sigma F_Y = 0$ at each joint
4. Solve for forces
5. Draw final FBD



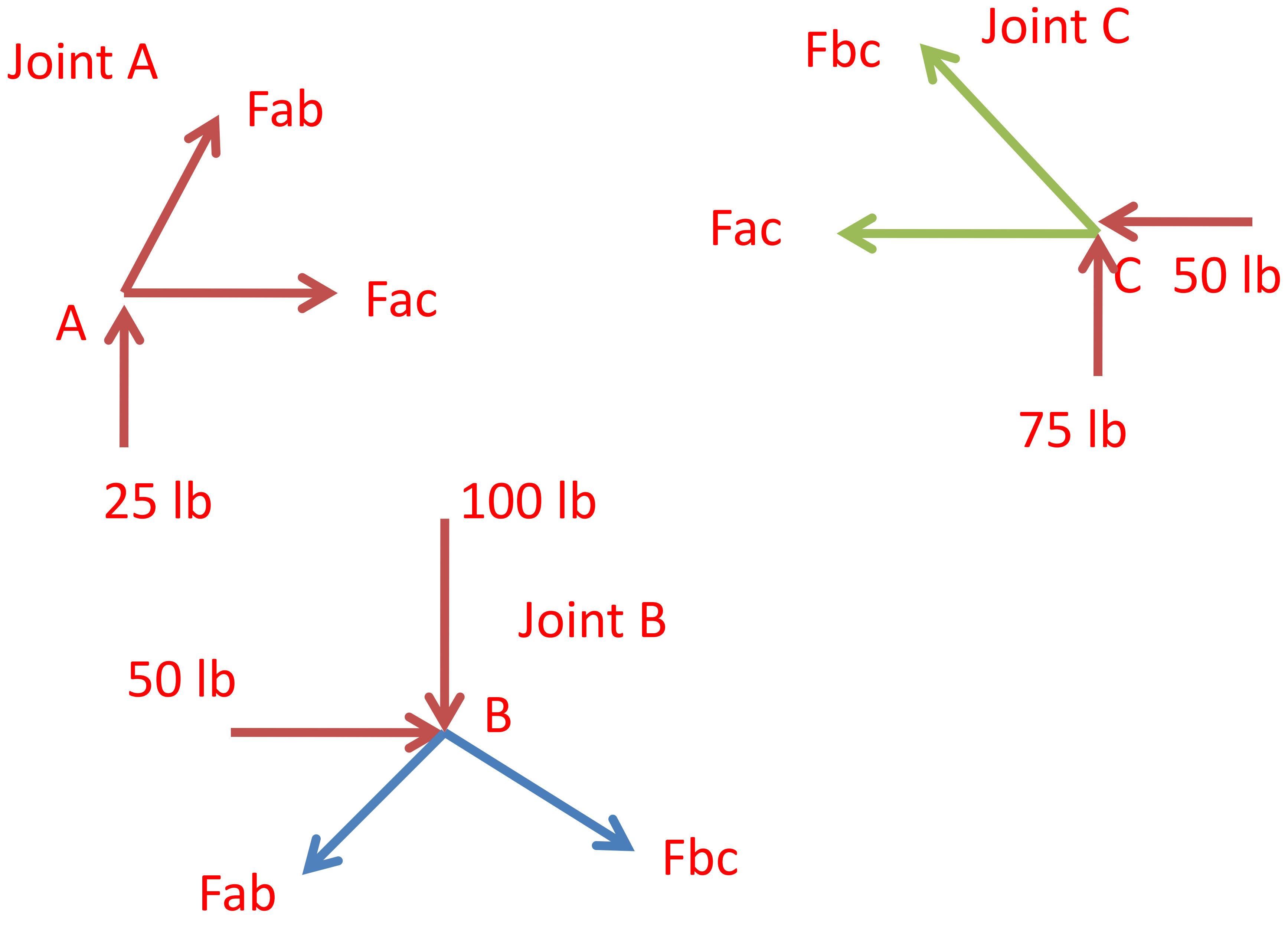
Truss Example
100.0
50.0
B
51.340°
68.199°
A
C
C
Ay
Cy

Truss FBD with solved Reaction Forces



Steps for finding Truss Forces

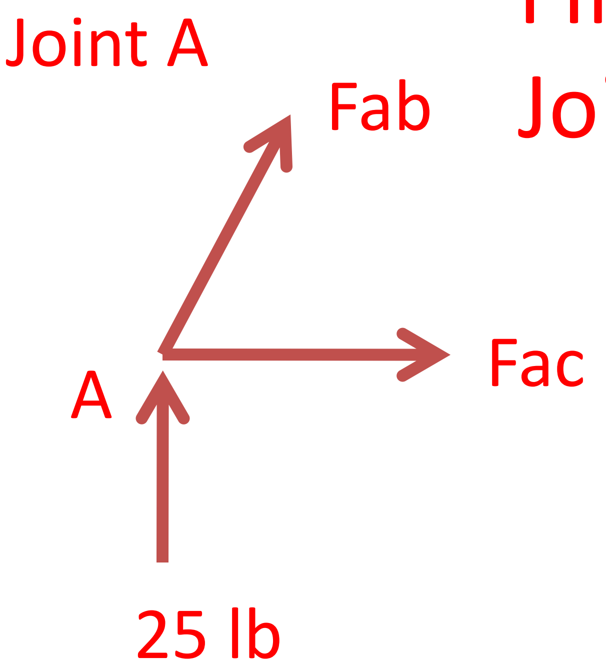
1. Solve for Reaction forces
   a. Draw a FBD of the entire system
   b. $\Sigma F_X = 0; \Sigma F_Y = 0; \Sigma M = 0$
   c. Use the above equations to solve for reaction forces
2. FBD of each joint (use vector properties)
3. $\Sigma F_X = 0; \Sigma F_Y = 0$ at each joint
4. Solve for forces
5. Draw final FBD

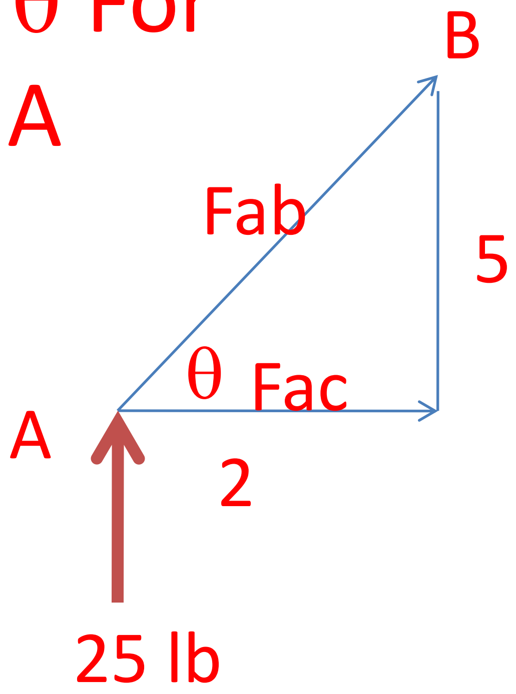

Joint A
Fab
Fac
A
25 lb
Joint C
Fbc
Fac
C
50 lb
75 lb
100 lb
Joint B
50 lb
B
Fab
Fbc

Steps for finding Truss Forces

1. Solve for Reaction forces
   a. Draw a FBD of the entire system
   b. $\Sigma F_X = 0; \Sigma F_Y = 0; \Sigma M = 0$
   c. Use the above equations to solve for reaction forces
2. FBD of each joint (use vector properties)
3. $\Sigma F_X = 0; \Sigma F_Y = 0$ at each joint
4. Solve for forces
5. Draw final FBD

FIND θ For Joint A





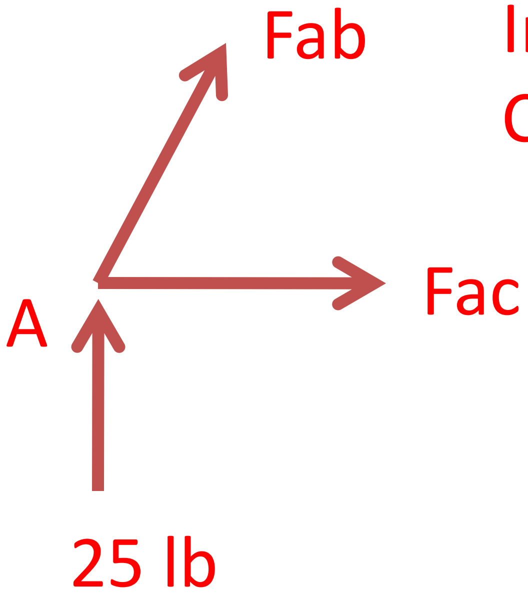
Find θ:

Tan θ = O/A

Tan θ = 5/2

$\theta = 68.2$ deg

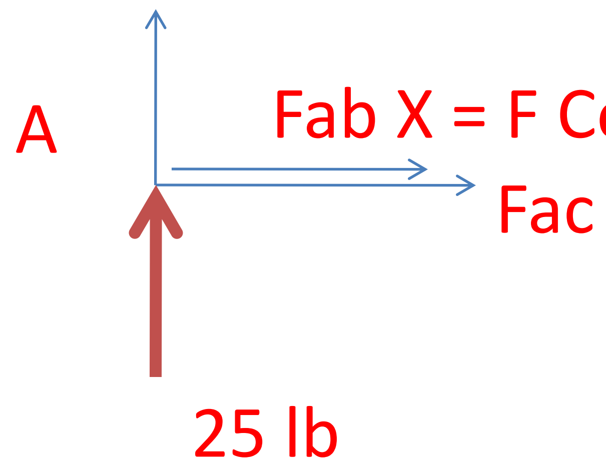
Joint A



Redraw Joint
In X and Y
Components

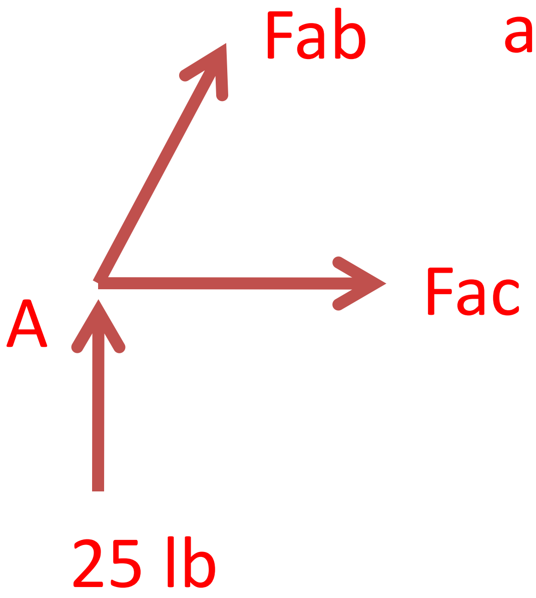
Fab Y = F sin θ = F sin 68.2

Fab X = F Cos θ = F cos 68.2



Joint A

Sum Forces in X and Y directions



Sum forces in y = 0

F sin 68.2 + 25 lb = 0

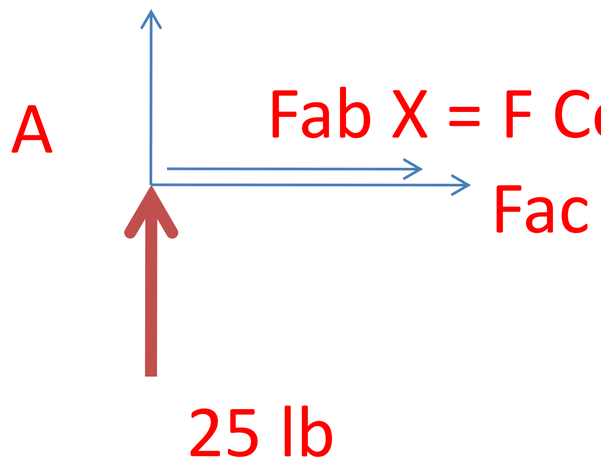
0.93 F = -25 lb

Fab = -26.9 lb

Fab = 26.9 lb in compression

$$\text{Fab Y} = F \sin \theta = F \sin 68.2$$

$$\text{Fab X} = F \cos \theta = F \cos 68.2$$



Sum forces in x = 0

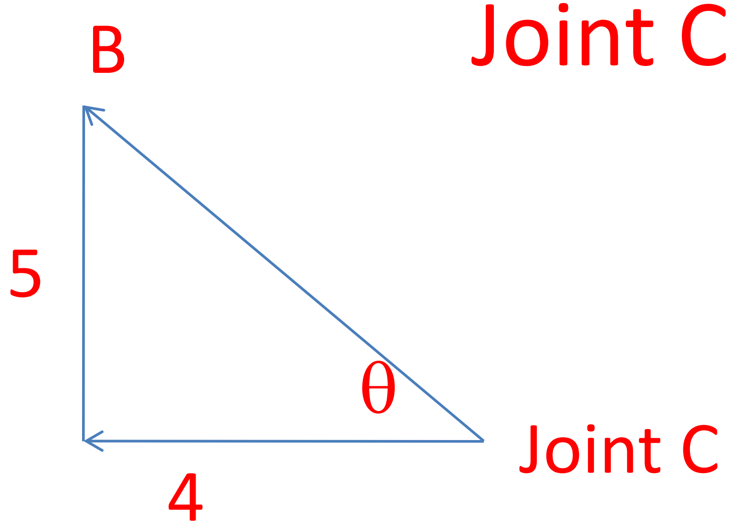
Fcos 68.2 + F ac = 0

-26.9 * cos 68.2 = -Fac

-10 = -Fac

Fac = 10 lb in tension

FIND θ For Joint C



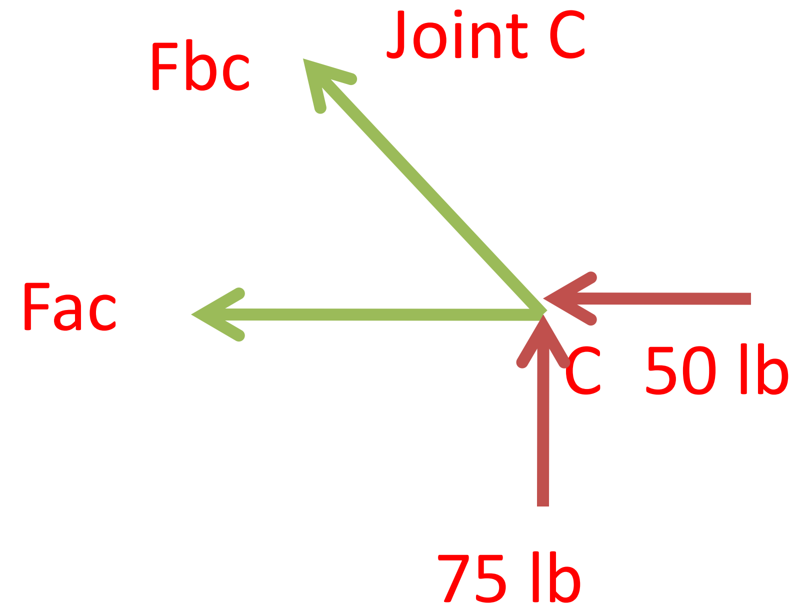
Find θ:

Tan θ = O/A

Tan θ = 5/4

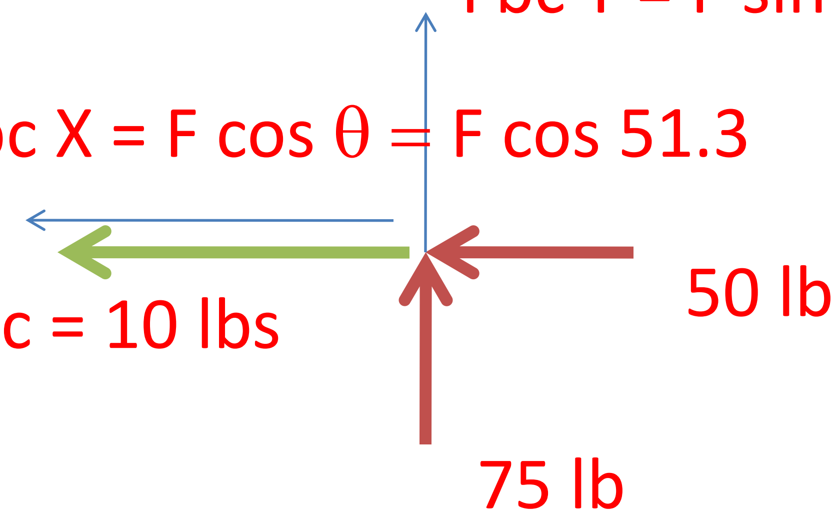
θ = 51.3 deg

Redraw Joint
In X and Y
Components



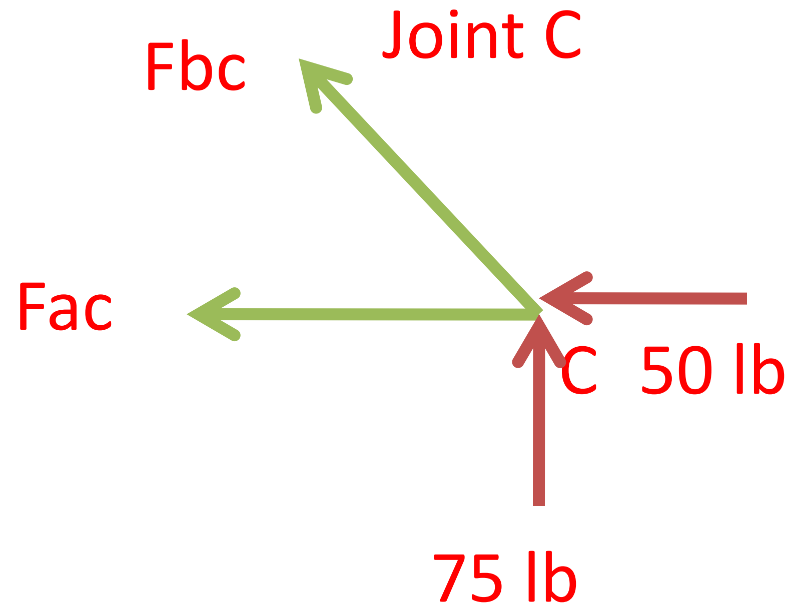
Fbc Y = F sin θ = F sin 51.3

Fbc X = F cos θ = F cos 51.3



Fac = 10 lbs

Sum Forces in X and Y directions



Sum forces in x = 0

-50 lb – 10 lb - Fcos 51.3 = 0

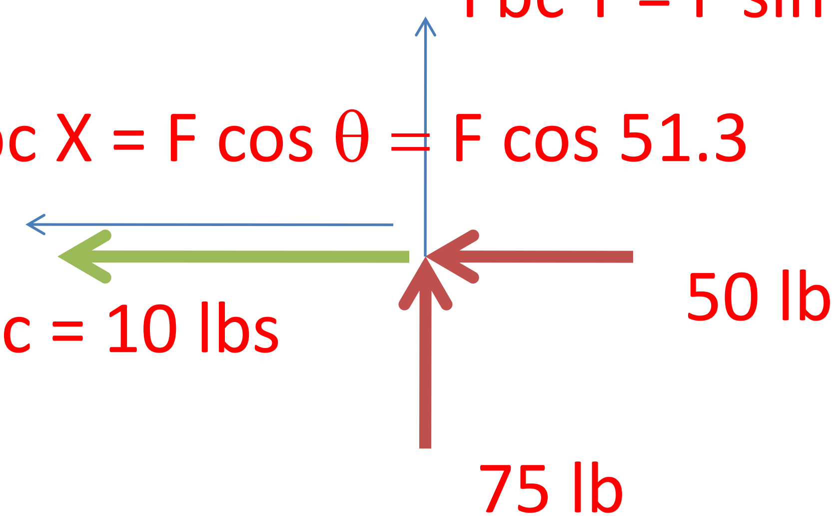
-60 lb = .625Fbc

-96 = Fbc

Fbc = 96 lb in compression

Fbc Y = F sin θ = F sin 51.3

Fbc X = F cos θ = F cos 51.3



Fac = 10 lbs

Sum forces in y = 0

F sin 51.3 + 75 lb = 0

0.78 F = -75 lb

Fbc = -96 lb

Fab = 96 lb in compression

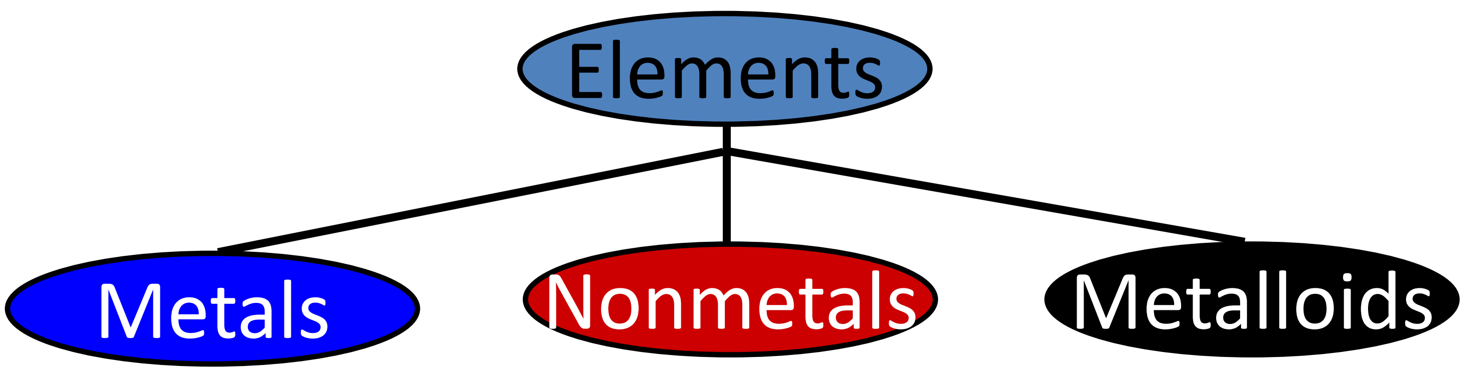
Material Properties

What Are Materials?

Materials: Substances out of which all things are made

What makes a material?

Materials are **consist of pure elements** and are categorized by physical and chemical properties



Material Composition - Elements

Metal Elements

Distinguishing Characteristics

Good conductors of heat and electricity, hard, shiny, reflect light, malleable, ductile, typically have one to three valence electrons

Material Composition - Elements

Nonmetal Elements

Distinguishing Characteristics

Most are gases at room temperature

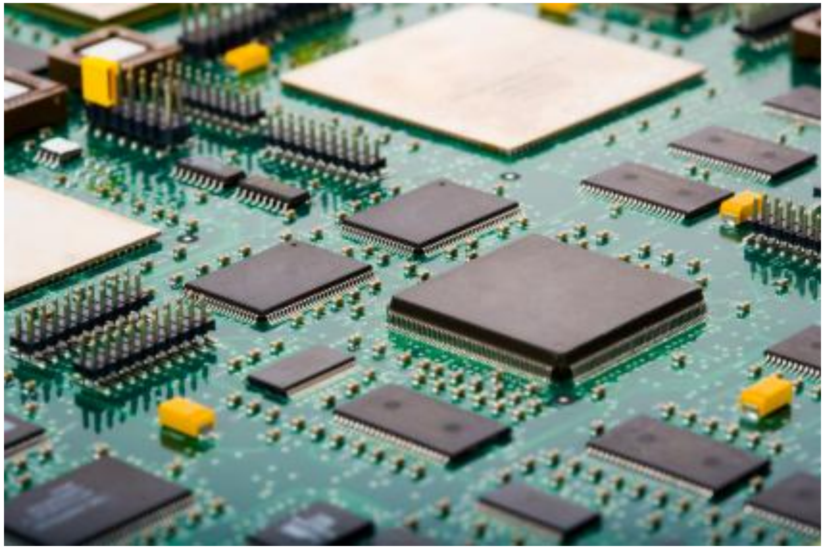
Solids are dull, brittle, and powdery; electrons are tightly attracted and restricted to one atom; poor conductors of heat and electricity

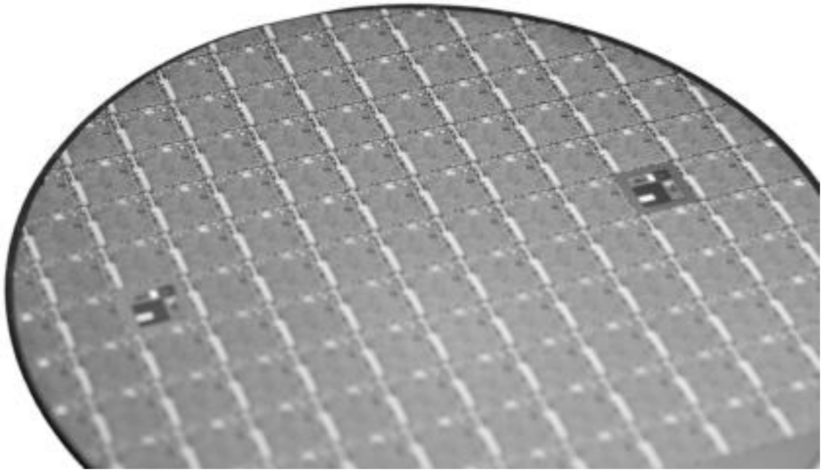
Material Composition - Elements

Metalloids

Distinguishing Characteristics

Possess both metallic and nonmetallic properties





Material Composition – Compounds and Mixtures

Compounds: created when two or more elements are chemically combined

Most substances are compounds

Mixtures: Non-chemical combination of any two or more substances

Elements within the mixture retain their identity

Material Classification

Common material classification categories:

- Metallic Materials
- Ceramic Materials
- Organic Materials
- Polymeric Materials
- Composite Materials

Metallic Materials

Distinguishing Characteristics

Pure metal elements
(Not commonly found or used)

Metal element compounds (alloy)
(Commonly used due to the engineered properties of the compound)

Thermal and electrical conductors

Mechanical properties include strength and plasticity

Ceramic Materials

Distinguishing Characteristics

Compounds consisting of metal and nonmetal elements

Thermal and electrical insulators

Mechanical properties include high strength at high temperatures and brittleness

Organic Materials

Distinguishing Characteristics

Are or were once living organisms

Consist of mostly carbon and hydrogen

Genetically alterable

Renewable

Sustainable

Polymeric Materials

Distinguishing Characteristics

Compounds consist of mostly organic elements

Low density

Mechanical properties include flexibility and elasticity

Polymeric Subgroups

Plastics

Elastomers

Composite Materials

Distinguishing Characteristics

Composed of more then one material

Designed to obtain desirable properties from each individual material

Material Selection

Refined material selection based upon:

Mechanical
Physical
Thermal
Electromagnetic
Chemical

Should also include recyclability and cost when choosing appropriate materials for a design

Material Selection

Mechanical Properties

Deformation and fracture as a response to applied mechanical forces

- Strength
- Hardness
- Ductility
- Stiffness

Material Selection

Thermal Properties

Affected by heat fluxes and temperature changes

Thermal Capacity – Heat storage capacity of a material

Thermal Conductivity – Capacity of a material to transport heat

Thermal Expansion – How a material expands or contracts if the temperature is raised or lowered

Material Selection

Electrical Properties

Material response to electromagnetic fields

Electrical Conductivity – Insulators, dielectrics, semiconductors, semimetals, conductors, superconductors

Thermoelectric – Electrical stimuli provoke thermo responses; thermo stimuli provoke electrical responses

Material Selection

Chemical Properties

Response and impact of environment on material structures

Oxidation and Reduction – Occur in corrosion and combustion

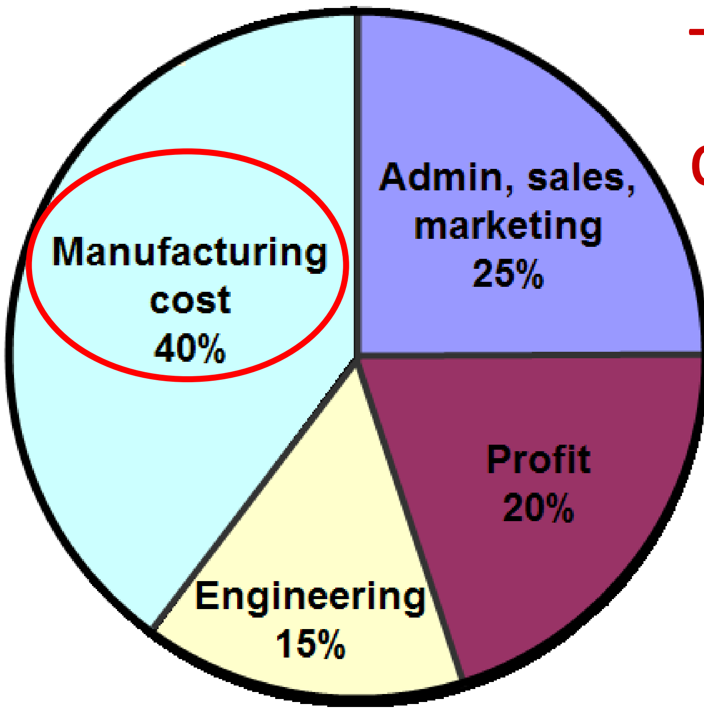
Toxicity – The damaging effect a material has on other materials

Flammability – The ability of a material to ignite and combust

Manufacturing Process

Product Creation Cycle

Design → Material Selection → Process Selection → Manufacture → Inspection → Feedback



Typical product cost breakdown

Manufacturing Processes

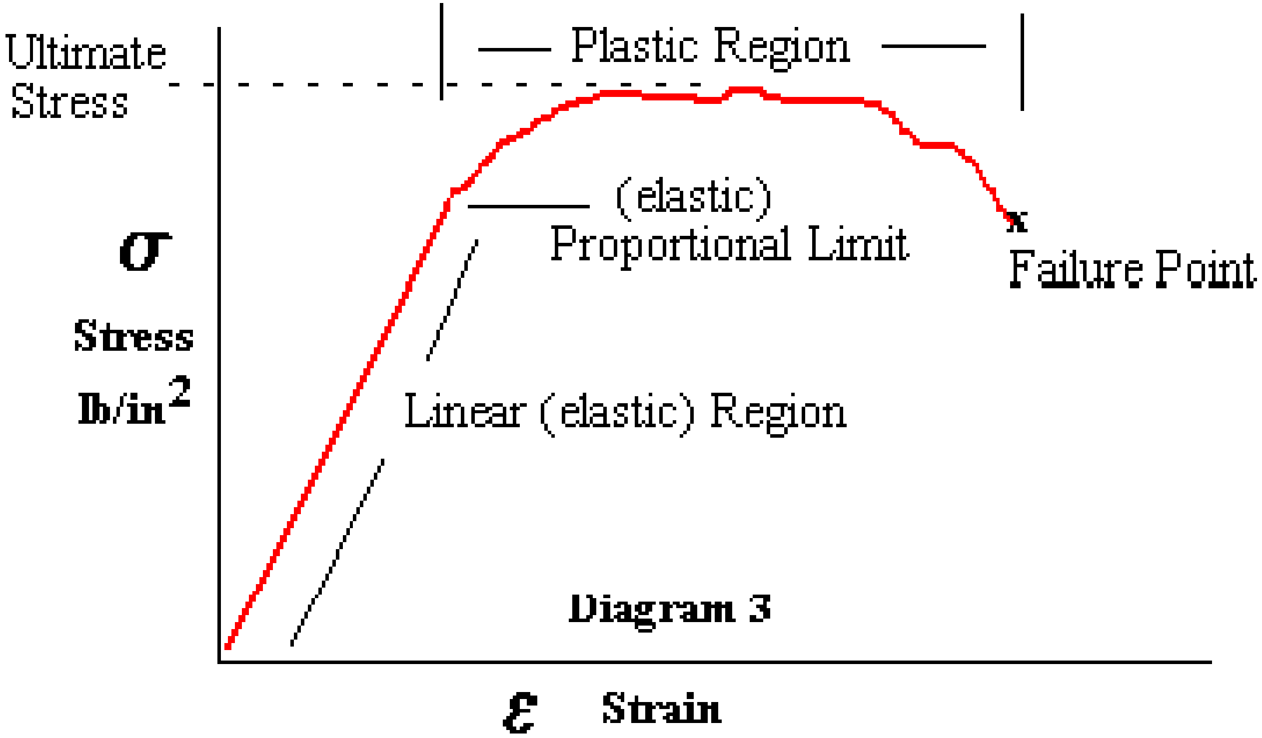
- Raw Materials undergo various manufacturing processes in the production of consumer goods

Material Testing

Material Testing

- Engineers use a design process and formulas to solve and document design problems.
- Engineers use destructive and nondestructive testing on materials for the purpose if identifying and verifying the properties of various materials.
- Materials testing provides reproducible evaluation of material properties

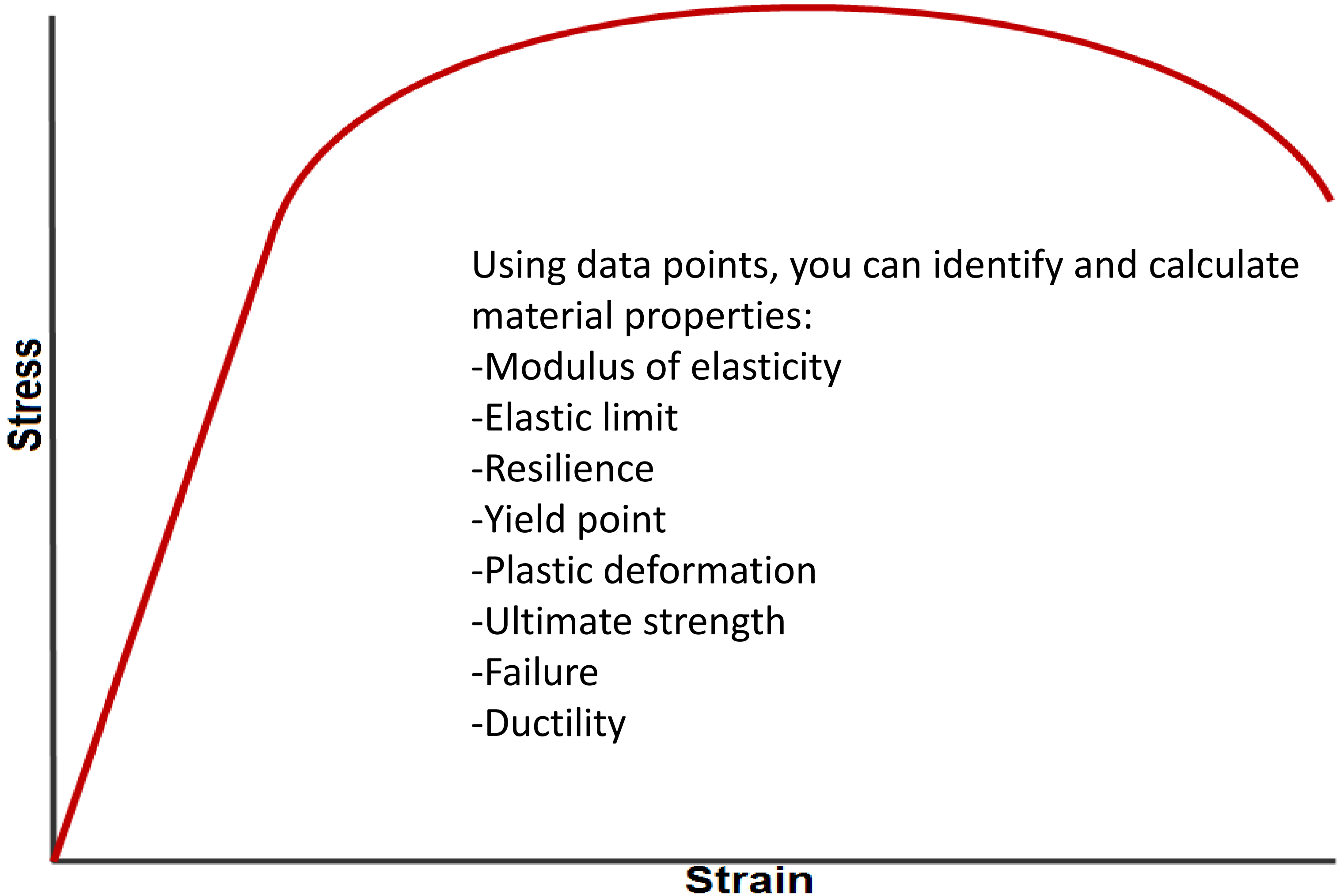
Stress- Strain Curve: created from tensile testing data



$$E = \frac{\sigma}{\varepsilon} = \frac{Stress}{Strain}$$

E is the Elastic Modulus.

E is the slope of the line in the elastic region.



Using data points, you can identify and calculate material properties:

-Modulus of elasticity

-Elastic limit

-Resilience

-Yield point

-Plastic deformation

-Ultimate strength

-Failure

-Ductility

Stress:
average amount of force exerted per unit area

Strain: a measurement of deformation in a structure due to applied forces.

- Strain is calculated from:

$$\text{Strain} = \frac{\text{Deformation}}{\text{Original Length}}$$

or

$$\varepsilon = \delta / L$$

- Strain is deformation per unit length, a dimensionless quantity

Proportional Limit: greatest stress a material is capable of withstanding without deviation from straight line proportionality between the stress and strain. If the force applied to a material is released, the material will return to its original size and shape.

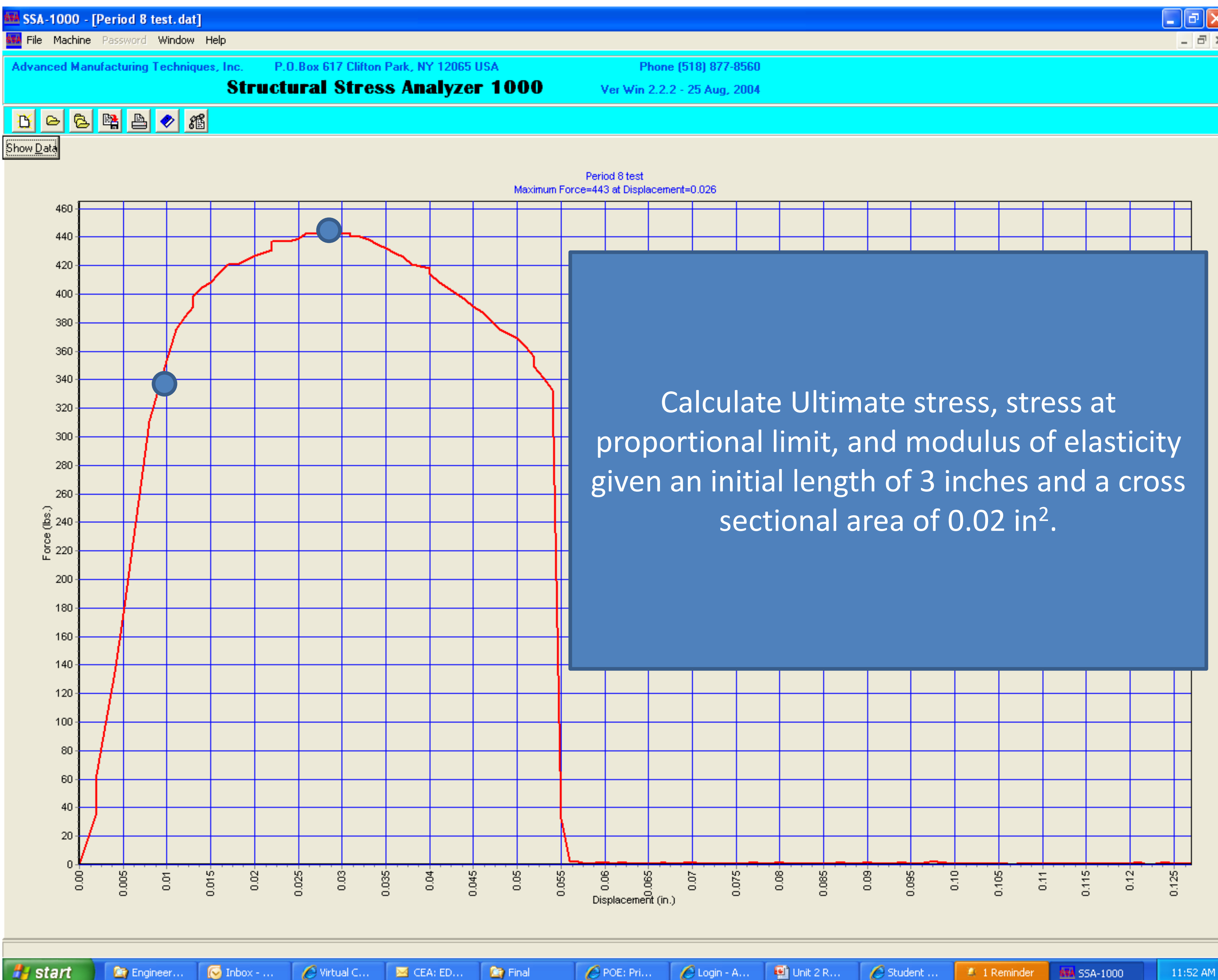
- **Yield Point:** The point at which a sudden elongation takes place, while the load on the sample remains the same or actually drops. If the force applied to the material is released, the material will not return to its original shape.

- **Ultimate Strength:** The point at which a maximum load for a sample is achieved. Beyond this point elongation of the sample continues, but the force exerted decreases.

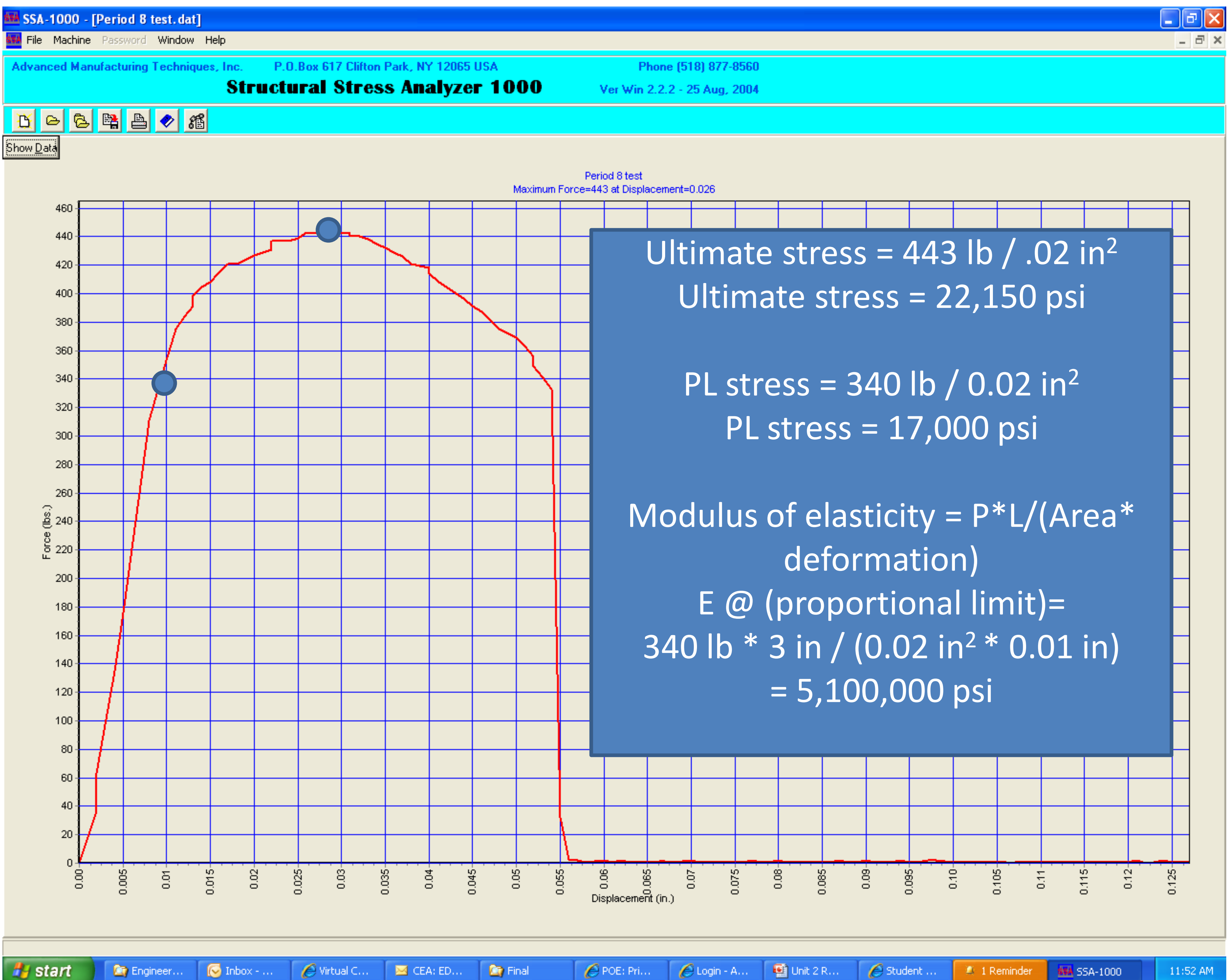
- **Modulus of Elasticity:** A measure of a material's ability to regain its original dimensions after the removal of a load or force. The modulus is the slope of the straight line portion of the stress-strain diagram up to the proportional limit.

- **Modulus of Resilience:** A measure of a material's ability to absorb energy up to the elastic limit. This modulus is represented by the area under the stress vs. strain curve from 0-force to the elastic limit.

- **Modulus of Toughness:** A measure of a material’s ability to plastically deform without fracturing. Work is performed by the material absorbing energy by the blow or deformation. This measurement is equal to the area under the stress vs. strain curve from its origin through the rupture point.



Calculate Ultimate stress, stress at proportional limit, and modulus of elasticity given an initial length of 3 inches and a cross sectional area of 0.02 in^2.



Ultimate stress = 443 lb / .02 in^2

Ultimate stress = 22,150 psi

PL stress = 340 lb / 0.02 in^2

PL stress = 17,000 psi

Modulus of elasticity = P*L/(Area* deformation)

E @ (proportional limit)=

340 lb * 3 in / (0.02 in^2 * 0.01 in)

= 5,100,000 psi

A 1” diameter piece of steel is 15 feet long. If the total tensile load in the steel is 125,000 pounds and the modulus of elasticity is 30,000,000 psi, calculate using the 5 step engineering process:

a) The tensile stress-
b) The total elongation caused by the load-
c) The unit elongation-

A 1" diameter piece of steel is 15 feet long. If the total tensile load in the steel is 125,000 pounds and the modulus of elasticity is 30,000,000 psi, calculate using the 5 step engineering process:

a) The tensile stress-
b) The total elongation caused by the load-
c) The unit elongation-

- Stress = P/A = 125,000 lbs/ (pi* 0.5 in* 0.5in) = 159,155 psi
- Elongation = P*L / (A*E) = 125,000 lbs * 15 feet / (pi* 0.5 in* 0.5in* 30,000,000 psi) = 0.08 ft or 0.96 inches.
- Unit Elongation is Strain, or deformation divided by length. =0.08 feet/15 feet = 0.00533

A 2” by 6” rectangular steel beam is 60 feet long and supports an axial load of 15, 000 lbs. Calculate using the 5 step engineering process:

a) The maximum unit tensile stress in the rod.

b) The maximum allowed load (P) if the unit tensile stress must not exceed 20,000 psi.

c) The total elongation if E= 30,000,000 psi using the maximum allowed load from part B.

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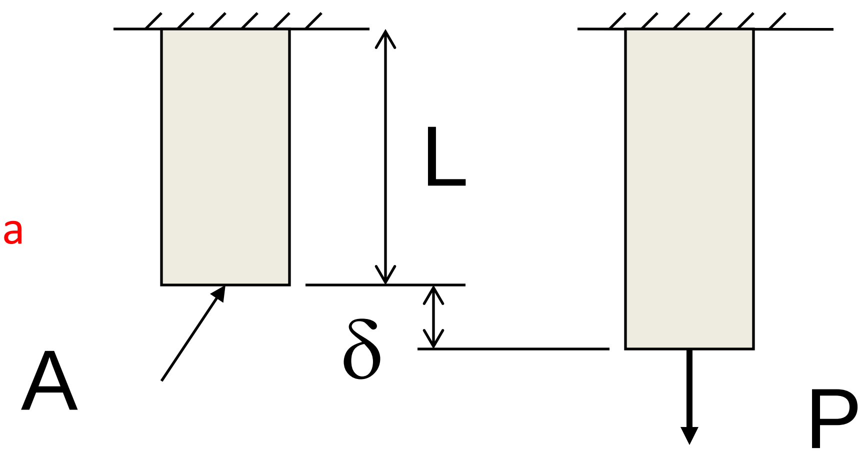
Area = 2” * 6” = 12 in^2

- Stress = P/A = 15000 lbs /12in^2 = 1,250 psi
- Stress = P/A 20,000 psi = P/12 in^2 P = 240,000 lbs
- Elongation is (P*L)/(A*E) = 240,000 lbs * 60 feet / (12 in^2 * 30,000,000 psi) = 0.04 feet or 0.48 in.

Deflection of Rod under Axial Load

$$\delta = \frac{P * L}{A * E}$$



Deflection is measure of the deformation in a structure.

Where: P is the applied load

L is the length

A is the cross section area

E is the elastic modulus

Stress/ Strain Example 1

A sample of material is ¼"diameter and must be turned to a smaller diameter to be able to be used in a tensile machine. The target breaking point for the material is 925 pounds. The tensile strength of the material is 63,750 psi. What diameter would the sample have to be turned to in order to meet the specified requirements?

Stress/ Strain Example 1

Knowns:

Load = 925 lb

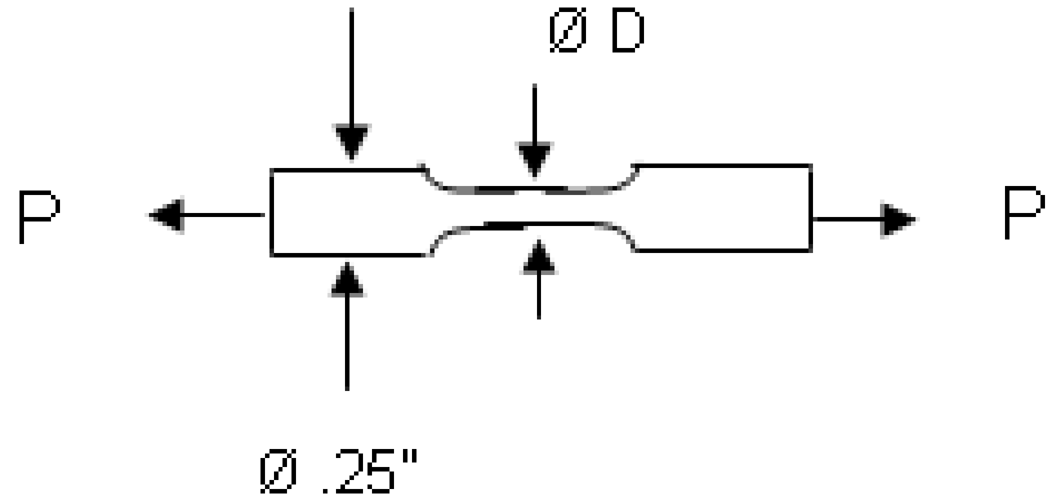
Stress = 63,750 psi

Unknowns:

Dia final = ?

Stress/ Strain Example 1

Drawing:



Equations:

- A = $\frac{\Pi D^2}{4} = .7854D^2$ = $\frac{P}{A}$

Stress/ Strain Example 1

Substitution:

63750 psi = $\dfrac{925lbs}{.7854D^2}$

Solve:

D^2 = $\dfrac{925lbs}{(.7854in)(63750psi)} \approx .018454413in^2$

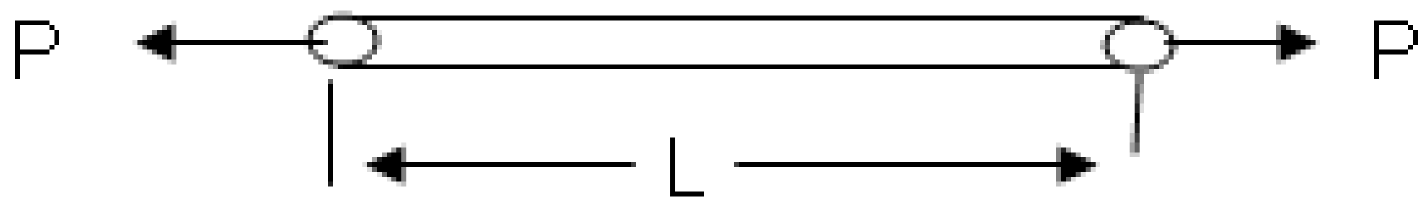
D = $\sqrt{0.18454413in^2}$ = 0.136”

Stress/ Strain Example 2

A strand of wire 1,000 ft. long with a cross-sectional area of 3.5 sq. inches must be stretched with a load of 2000 lb. The modulus of Elasticity of this metal is 29,000,000 psi. What is the unit deformation of this material?

Stress/ Strain Example 2

Drawing:



Equations:

$\delta = \dfrac{PL}{AE}$ $\varepsilon = \dfrac{\delta}{L}$

Stress/ Strain Example 2

Knowns:

L = 1000’ = 12000”   A = 3.5 in^2

P = 2000 lb   E = 29 x 10^6 psi

Unknowns:

Unknowns: ε

Stress/ Strain Example 2

Substitution/Solve:

$$\delta = \frac{PL}{AE} = \frac{(2000lb)(12000in)}{(3.5in^2)(29x10^6\,psi)} = 0.236 \text{ in}$$

$$\varepsilon = \frac{\delta}{L} = \frac{.236in}{12000in} = 0.0000197\text{in}$$